A MODEL FOR THE OPTIMAL RISK MANAGEMENT OF FARM FIRMS

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Abstract

Risk management is an integrated part of business or firm management and deals with the problem of how to avoid the risk of economic losses when the objective is to maximize expected profit. This paper will focus on the identification, assessment, and prioritization of risks in agriculture followed by a description of procedures for coordinated and economical application of resources to control the probability and/or impact of unfortunate events. Besides identifying the major risk factors and tools for risk management in agricultural production, the paper will look critically into the current methods for risk management. Risk management is typically based on numerical analysis and the concept of efficiency. None of the methods developed so far actually solve the basic question of how the individual manager should behave so as to optimise the balance between expected profit/income and risk. In the paper, we derive a criterion for optimal risk management in the sense that we derive the optimal combination of expected income and variance on return to capital on the efficient frontier.

Key words: expected income, profit maximisation, risk, uncertainty, variance

INTRODUCTION

Risk management is an integrated part of business or firm management which deals with the problem of how to avoid the risk of economic losses when the objective is to maximize expected profit. Historically, the methods used for risk management have to a large extent been based on the Expected Utility model [17] and the Modern Portfolio Theory based on the work of Markowitz [11]. Markowitz’s approach was to use linear programming method to find the combination of investments (portfolio) that would provide the efficient combination of risk and return to capital. Since the World War II, along with the development of the computer technology, Monte Carlo simulation has become an important tool for (numerical) risk analysis. Another important contribution to the development of tools for risk management are the concept of stochastic dominance, which is based on the work of Hadar and Russell [4] and Hanoch and Levy, and Option Theory which is based on the work of Black and Scholes [1]. Additional methods such as Value-at-Risk [9], have evolved from Modern Portfolio Theory, Option Theory, and the concept of stochastic dominance, but none of these methods are significant improvements of the original earlier methods.

Hubbard [8], provides a good review of the theoretical and methodological development of risk management since World War II. He also provides a formal definition of risk management being: ‘The identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events’ or simply ‘Being smart about taking chances’ (ibid, p. 10). Chapman [2] provides an extensive introduction to the state-of-the-art regarding applied business risk management based on the ISO 31000 standards published by the International Organization for Standardization [14]. None of the methods which have been developed so far actually solve the basic question of how the individual manager should behave (carry out risk management) so as to optimize the balance between expected profit/income and risk. Current methods for risk management are typically based on numerical analysis and the concept of efficiency, and operational criteria for optimal risk management do not exist.
In this paper we use the theory of financial economics to derive a criterion for \textit{optimal risk management} of firms. The criterion is derived using the Capital Asset Pricing Model which provides an economic based vehicle for analytical derivation of the trade-off between risk and expected income/profit.

\section*{BACKGROUND}

The international literature reveals that the objective of risk management is difficult to describe in operational terms. Efficiency seems to be the key term, and the objective of risk management may be stated as a way to manage the business in an \textit{efficient way}, i.e. to minimise risk for a given level of the other objectives (profit/income) or to maximise the achievement of other objectives (profit/income) for a given level of risk. According to a Draft International Standard from the International Organization for Standardization [14] risk management \textquotedblleft…aids decision making by taking account of uncertainty and its effect on achieving objectives and assessing the need for any actions” (p. V), and risk management \textquotedblleft…should thus help avoid ineffective and inefficient responses to risk that can unnecessarily prevent legitimate activities and/or distort resource allocation” (p. VI).

In agricultural economics (farm management), risk management has historically been based on the Expected Utility (EU) model and various ad hoc procedures based on stochastic simulation and the concept of stochastic dominance and Value-at Risk.\footnote{Hardaker (2006) provides a good review of the approaches to risk management in agriculture and the challenges ahead.} However, none of the methods used so far actually takes the normative approach in the sense that they describe how in fact to balance the cost of implementing each risk management option against the benefits derived from it. In general, the cost of managing risks needs to be commensurate with the benefits obtained. For instance, threshold levels of probabilities of loss used in the Value-at-Risk method do not provide an objective measurement commensurate with the cost of implementing risk management tools.

Although maximization of expected utility (EU) has become the standard paradigm for risk analysis, the basic problem faced by the researcher namely that the decision maker’s utility function normally is unknown, still exists. Under certain assumptions it is possible to quantify expected utility \textit{U} as: \( U = E(w) - 0.5^*R_A*V(w) \), where \( w \) is wealth, \( V \) is variance, \( R_A \) is absolute risk aversion coefficient \( R_A = -v''(w)/v'(w) \), \( v \) is the Von Neumann-Morgenstern utility function, and \( E \) is the expected value operator [3, 7]. This functional form of the utility function is the closest the theory of risk management has been to an operational model on which to objectively base risk management decisions [13, 15]. However, even though this model seems operational as an optimization model it is not the case because the risk aversion coefficient \( R_A \) is not in general known.

In its report on international standards for risk management [14], the International Organization for Standardization mention as the first of its suggested principles for risk management, that risk management \textit{creates value}. This statement generated the idea, that instead of focussing on maximizing of expected utility, the approach the approach is changed to the maximizing the value of the firm. By changing the focus to the maximization of the value of the firm, the theory of financial economics including the Capital Asset Pricing Model (CAPM) would provide a valuable tool on which to base the analysis of (optimal) risk management. This is the background for the model presented in the following.

But before the model is presented, I will give a short overview of risk and controlling of risk in agriculture.

\section*{RISK IN AGRICULTURE}

The main problem in risk management is to consider answering three questions: a) To what extent should the risk be controlled/reduced? b) What risk management tools should be used? c) In what order of priority should the risk management tools be applied?

In the literature on risk management in agriculture, the questions b) and c) have been widely discussed [4]. However, the literature
In principle one can choose between the following four major risk management strategies: 1) Keep the risk. 2) Avoid the risk. 3) Reduce the risk. 4) Transfer all or part of the risk to others. The first three of these four strategies relate to initiatives for risk management at the enterprise level, while the last relates to initiatives that go outside the company.

The four main strategies can be described as follows:
1. **Keep the risk**: Farmers can manage risk by tackling it herself, e.g. by having sufficient cash reserves in the form of savings, financial reserves or ability to obtain credit. This makes the farmer/farmer family able to maintain consumption as income or capital loss are avoided by funding from cash reserves. This form of self-insurance is the most efficient form of risk management when there are transaction costs of using other alternatives such as formal insurance. Use of cash reserves as self-insurance implies, however, in itself costs in terms of opportunity costs of holding liquid capital. Therefore, the final choice is ultimately dependent on the "insurance form" which is cheapest.

2. **Avoid the risk**: Farmers can (try to) avoid the risk by choosing products and production methods that are not (so) vulnerable to non-controllable events such as weather, disease and unstable prices. Examples of such initiatives in agriculture, e.g. choice of safe crops, selection of appropriate crop rotations, choice of products with stable prices, selection of productions with short production, cultivation of crops under controlled conditions (greenhouse, irrigation, drainage facilities), etc.

3. **Reduce the risk**: Actions to reduce risk include both reduction of the impact/-consequences of the uncertain factors, and reducing the likelihood of loss. Spread of activities (diversification), including income from sources other than agriculture, production monitoring, pest control, disease control, flexibility (plant flexibility, product flexibility, market flexibility), etc. are examples of such concrete actions that can be used to reduce risk.

4. **Transfer all or part of the risk**: The transfer of risk to others (or sharing the risk with others) mainly comprises insurance in the traditional
sense. In addition, a number of financial instruments, etc. with a function similar to insurance. Finally, the risk transfer achieved by the conclusion of contracts with others on the production and marketing.

**CONTROLLING RISK**

One can distinguish two main types of risk management instruments: 1) Target (market) instruments for risk management and 2) general (internal) methods for managing and controlling risk.

1. Market-based risk management instruments. The market based instruments are characterized by the common feature that there is a price attached to them, and one can readily calculate the cost of their use. The market-based risk management instruments are:
   a) *Weather derivatives* for protection against production risk in crop production.
   b) *Forward contracts* to protect against market risk (price risk).
   c) *Futures* to protect against market risk (price risk) and financial risk.
   d) *Put options* to protect against market risk (prices of products).
   e) *Call options* to protect against market risk (price increase in raw materials).
   f) *Interest rate swaps* to protect against financial risk (interest rate increase).
   g) *Insurance* (fire, storm, hail, etc.) to protect against the risk associated with production.
   h) *Product Liability Insurance* to protect against product risks.

2. General (internal) risk management instruments. The general methods of risk management include changes in its financial and manufacturing organization in order to control risk. The cost of these tools must be estimated using the opportunity cost principle. These instruments include the following:
   j) Financial reserves/equity. Maintaining own (cash) funds or collateral provisions, including withholding of dividends.
   k) *Product Selection*. Choice of products (crops) with safer yield, shorter production period, etc.

   1) **Production Methods**: Safer production systems, crop rotation, disease prevention, the use of greenhouses, irrigation systems, drainage systems etc.
   m) **Diversification**. Spread of activities.
   n) **Monitoring/control**. Monitoring of production, pest control, disease control.
   o) **Flexibility**. Plant flexibility, liquidity facilities, product flexibility and market flexibility.

**THE MODEL**

The risk management model derived in the following is based on the classical assumption that the goal of firm management is to maximize profit measured as the market value of the firm’s equity capital. For a shareholder company this means that the goal is to maximise the market value of the company’s shares. For the individually owned firms this means that the goal is to maximise the net value of the firm (i.e. market value of the firm minus the market value of debt). This is in accordance with Chapman [2], who defines (enterprise) risk management as “a comprehensive and integrated framework for managing company-wide risk in order to maximise a company’s value”.

The relationship between the *expected return* to equity, the *risk on return* to equity and the *market value* of equity can be analysed within the framework of the Capital Asset Pricing Model (CAPM) [16] and [10, 12.]

According to the CAPM, a company that still wants to attract equity to finance its business must yield an expected return $E(R)$ to equity of at least $R_F + \beta(E(R_M) - R_F)$, where $E$ is the expected value operator, $R_F$ is the risk-free rate of interest (for instance the rate of return on government bonds), $R_M$ is the interest on the market portfolio (i.e. the rate of return on a well-diversified portfolio of assets in which the potential investor would alternatively invest her money) and $\beta$ is a risk factor defined as:

$$\beta = \frac{C(R, R_M)}{V(R_M)} \quad (1)$$

where $C(R, R_M)$ is the covariance between $R$ and $R_M$, and $V(R_M)$ is the variance of $R_M$. 

21
This means that in equilibrium:

\[ E(R) = R_f + \beta(E(R_M) - R_f) \]  

(2)

Equation (2) may be used to determine the market value of the equity of the firm. If the firm has an expected earnings before interest of \( E(a) \), a total debt of \( B \) on which it pays a fixed interest \( R_B \), then the expected earnings to shareholders \( E(c) \) is \( E(c) = E(a) - R_B B \). To provide an expected return to equity of \( E(R) \), the market value \( S \) of equity has to be:

\[ S = \frac{E(c)}{E(R)} \]  

(3)

In the CAPM, the relationship between returns to equity \( R \) and the risk of returns to equity, quantified as the variance \( \sigma \) of returns to equity \( V(R) \), is determined through the risk factor \( \beta \). The first step therefore is to consider the covariance between the variance of returns to equity \( V(R) \) and the variance \( C(R, R_M) \) of market returns.

To do this, the following result will prove useful:

**Proposition 1:** If increasing (decreasing) variance is interpreted as being a proportional increase (decrease) in the deviations from the mean, then:

\[ \frac{dC(R, R_M)}{dV(R)} = \frac{C(R, R_M)}{2V(R)} \]  

(4)

which means that the change in the covariance between \( R \) and \( R_M \) when the variance of \( R \) changes, is equal to the covariance between \( R \) and \( R_M \) divided by two times the variance of \( R \).

**Proof of proposition 1:** First multiply all deviations \( (R_i - E(R)) \) \( (i = 1 \ldots T) \) by a constant \( \lambda \) after which the variance of \( R \) can be expressed as:

\[ V(R) = V(\lambda R) \bigg| \lambda = 1 = \lambda^2 V(R) \bigg| \lambda = 1 \]  

(5)

In a similar way, the covariance between \( R \) and \( R_M \) can be expressed as:

\[ C(R, R_M) = C(\lambda R, \lambda R_M) \bigg| \lambda = 1 = \lambda C(R, R_M) \bigg| \lambda = 1 \]  

(6)

The derivative of \( C(R, R_M) \) with respect to \( \lambda \) (at the point \( \lambda = 1 \)) is therefore equal to \( C(R, R_M) \), and the derivative of \( V(R) \) with respect to \( \lambda \) is 2V(R). Division of the derivative of \( C(R, R_M) \) with respect to \( \lambda \) by the derivative of \( V(R) \) with respect to \( \lambda \) produces the result in equation (4). (QED).

Using proposition 1, it is possible to show that the change in the risk factor \( \beta \) in (1) caused by a change of \( V(R) \) is:

\[ d\beta = \frac{C(R, R_M)}{V(R)(R_M)2V(R)}dV(R) \]  

(7)

and the corresponding change in the required expected returns to equity is therefore (see (2)):

\[ dE(R) = \frac{C(R, R_M)}{V(R)(R_M)2V(R)}dV(R)(E(R_M) - R_f) \]  

(8)

The change of the variance of the return to equity \( dV(R) \) on the right hand side of equation (8) thus changes the demand for the return to equity \( dE(R) \) on the left hand side of equation (8), which - according to equation (3) - implies a change in the market value of equity \( S \). At the same time, the change of the variance of return to equity (accomplished through risk management) implies change in costs, so that the expected earnings to shareholders \( E(c) \) changes. This will also - according to equation (3) - imply changes in the market value of equity \( S \).

The optimal effort of risk management is the effort that maximises the market value of equity. This is the case when the two contributions \( dV(R) \) and \( dE(c) \) balances each other, i.e. have the same (but opposite) influence on the market value of equity, so that the change in the value of equity for marginal changes of risk management is zero. The formal derivation is the following:

The market value of equity \( S \) is (see equation (3)) a function of \( E(c) \) and \( E(R) \). The total differential of this function is:

\[ dS = \frac{\partial S}{\partial E(c)} dE(c) + \frac{\partial S}{\partial E(R)} dE(R) \]  

(9)

The optimal effort of risk management is accomplished by maximising the equity \( S \) which – under certain conditions – is the case when \( dS \) in (9) is zero. Setting the right hand
side in (10) equal to zero and solving for \( dE(c) \) yields:

\[
dE(c) = -\frac{\partial S}{\partial E(R)} \cdot dE(R) - \frac{\partial S}{\partial E(c)} \cdot dE(c)
\]

(10)

If the differentiation of \( S \) (see (3)) is carried out, the equation becomes:

\[
dE(c) = \frac{E(c)}{E(R)} \cdot dE(R)
\]

(11)

Now insert the expressions of \( E(R) \) (from (2)) and \( dE(R) \) (from (8)) in equation (11), then the condition (11) may be expressed as:

\[
dE(c) \cdot V(R) = \frac{1}{2} \left( \beta (E(R_{d}) - R_{p}) \right)
\]

(12)

The optimal level of risk management is therefore determined by the condition that the relative change of the expected earnings to shareholders \( (E(c)) \) divided by the relative change in the variance of the return to equity \( (V(R)) \) is equal to the value of the term on the right hand side of equation (12).

It is reasonable to assume that the feasible combinations of \( E(c) \) and \( V(R) \) is a convex set as illustrated by the set \( P \) in Fig. 1.

As the market value of equity increases in \( E(c) \) and decreases in \( V(R) \) the optimal solution will be at the efficient frontier of \( P \). If the present position of the firm is for instance the efficient point \( A \) and this point is not optimal because the left-hand side of equation (12) is different from the right-hand side of equation (12), then the relationship between expected earnings and return to equity should be adjusted through risk management.

If the left-hand side of (12) is larger than the right-hand side, then the firm should accept greater risk by increasing expected earnings. If the left-hand side of (12) is less than the right-hand side, then the firm should reduce risk and accept less expected earnings. Further risk management is carried out as long as the left-hand side in (12) is different from the right-hand side. Assume that the value on the right and left-hand sides of (12) become equal at the

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Fig. 1. Risk management from A to O
point \( O \). In this case the optimal combination of expected earnings and variance is \( E^*(c) \) and \( V^*(R) \).

**CONCLUSIONS**

Only the necessary conditions for optimal risk management have been derived. The further condition is that risk management is carried out efficiently, i.e. that risk is reduced with a minimum reduction in expected income, and that an increase in expected income is achieved with a minimum increase in risk. However, this may be insufficient. Condition (12) may, in practice, have no solution or possibly as many as two depending on the available risk management tools. In the case of two solutions, the sufficient condition is that the left-hand side of (12) is decreasing in \( V(R) \).

The model is valuable in the sense that it provides a relatively simple, objective and operational method for evaluating risk management effort. Its weakness is that it is based on the assumption that all relevant information is available in the trade-off between expected income and variance. However, this simplification is also present in the CAPM model which has been an extremely valuable tool in empirical work and practice (Brounen, de Jong and Joedijk, 2006).

**REFERENCES**